Exercise Digital Image Processing

SS 2008

Exercise 5
Submit by May, 26th, 10:00AM, for exercise on May, 28th

MULTIMEDIA COMPUTING LAB
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Notes:

- You have a choice: Solutions to text exercises that do not involve programming can be in English or German, at your choice. Submission to text exercises can be made on paper or by email (scanned documents, PDFs, or Word/OpenOffice) to Eva.Hoerster@informatik.uni-augsburg.de before the above due date.
- Solutions to programming exercises must be submitted by email to eva.hoerster@informatik.uni-augsburg.de before the above due date. Only submit your source code (*.h and *.cpp files). Do not submit any executables, binary or object files, project or solution files, nor any other input data that can be downloaded from the course website (i.e. image or video data provided as part of the assignment). DO NOT COMPRESS YOUR SOURCE CODE FILES (.rar, .zip, etc. is not allowed)! Your code must compile and run; if your code fails to compile, you will receive zero points for the exercise.

5.1 (65 points)
In this exercise you should write a program that implements the 1D-DFT in C/C++. The 1D Fourier Transform maps an ordered N-dimensional vector \( g = [g_0, g_1, \ldots, g_{N-1}]^T \) of complex numbers onto another complex vector \( f = [f_0, f_1, \ldots, f_{N-1}]^T \) of the same dimension:

\[
f_v = \frac{1}{N} \sum_{n=0}^{N-1} g_n \left[ \cos \left( -\frac{2\pi n v}{N} \right) + i \sin \left( -\frac{2\pi n v}{N} \right) \right]; \quad 0 \leq v < N
\]

The back transform is given by:

\[
g_n = g_0 + g_1 + \ldots + g_{n(N-1)}; \quad 0 \leq n < N
\]

where

\[
g_{nv} = f_v \left[ \cos \left( \frac{2\pi n v}{N} \right) + i \sin \left( \frac{2\pi n v}{N} \right) \right]; \quad 0 \leq v < N
\]

a) Write a function that takes a complex vector of length \( N \) and computes its DFT transform by the equation given above.

b) Write a function that takes a complex vector of length \( N \) and computes its back transform by the equation given above.
c) Write a function that appropriately displays a given complex vector of length N. Display the real and the imaginary part in two separate windows. You may only consider the case where N=16. An example of how to display such a vector is given on slide 22 (‘First 9 Basis Functions’) of Lecture 4.

d) Now assume that N=16. Write a main function that asks the user to provide the values for a 16 dimensional input vector, displays the input vector and computes its DFT transform. Now we want to observe the single ‘steps’ of the back transform. Therefore display the results after each summation. I.e. display $g_{n0}$, then $g_{n0} + g_{n1}$ etc., until the complete input vector $g_n$ is reconstructed.

e) What do you observe if the input vector is real, i.e. its imaginary parts are all equal 0?

5.2 (35 points)
Show that the basis matrices of the two-dimensional DFT are orthogonal to each other. How many basis matrices are there for an image of size width w x height h?

Hint: The equation of the basis matrices is shown on slide “2D-DFT” of Lecture 4.